Circuit Model of a Phase-Shifting Transformer for Analog Power Flow Emulation

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Abstract— **This paper focuses on the development of a phase shifting transformer model designed to be utilized in analog simulation (emulation). Simulation of power systems is necessary for both operation and planning and depends on suitable models of the components in the system. Analog computation as a technology for the analysis of large scale power systems represents a viable alternative to traditional digital methods as it provides certain advantages, such as speed and parallelism. Prior research in this field has modeled generators, loads and transmission lines. The transformer model proposed here provides a more accurate depiction of the network and captures the behavior of phase shifting transformers. The transformer is modeled in analog form. The circuit model is constructed and verified via software simulation using Analog Behavioral Models (ABMs) in PSpice and compared with industrial grade numerical simulator for validation.**

I. INTRODUCTION

Depending on the impedances and the voltage at the terminals of transmission lines in the network, power flows on multiple paths. Conventional ac power systems lack controllability over active and reactive power flows in the transmission network and most of the control systems, such as excitation control and automatic governor control take action at the generating units. Current transmission networks are utilized and stressed to their limits and to ensure that operational conditions of the networks are maintained, power flow control within the network are evident. One of these solutions is a phase-shifting transformer. Besides the ability to control active power flow, phase-shifting transformers can increase operation flexibility and network security, and reduce losses in regional networks. Different types of phase shifting transformers have been installed or commissioned to serve for the purpose of relieving stress in small pockets of big loads with no generation. As an example, a 300 MW Variable Frequency Transformer (VFT) station has been commissioned in Linden, New Jersey, mainly for the purpose of transmitting controlled power into New York City [1]. The VFT is essentially a continuously variable phase-shifting transformer.

Power flow studies done via simulation of the power system are necessary for both planning and operations and depend on appropriate models of components in the system. In traditional digital simulation, computation for large scale power systems is time intensive as calculations are non linear in nature and lengthy iteration schemes such as Newton-Raphson and Gauss-Seidel are used. Solutions of the power flow studies using these schemes are largely dependent on size and complexity of the network increasing computational time. Analog computation as a technology for the analysis of large scale power systems has been reintroduced by Fried et al. [2]. This technology represents the performance of the physical system by solving the mathematical equations or scaled relationships defining the behavior of the system in a continuous manner. It represents a viable alternative to traditional digital methods as it provides certain advantages, speed and parallelism among them, and features in solving power system problems [3]:

- Variables that are complex in nature in power systems and that are usually represented in phasor form can easily be converted and expressed in rectangular form, which is more efficiently represented in analog circuits;
- Mathematical equations that describe the states and behaviors of power system under various conditions can be represented using analog devices and analog building blocks (e.g., summing, integration, multiplication, etc.);
- The representation of a power system can be scaled in time and amplitude. With proper choice of time-scale factor, the speed of analog computation of any size of power system can be made faster or slower than real-time.

This paper is an extension of the author's previous work [4] on circuit oriented analog modeling and it incorporates cross coupling dependent sources [5] that enhance the functionality of the phase-shifting transformer model used for this computation scheme. Additionally the model presented here when lumped in the transmission network is suitable for

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use in power flow analysis. In this paper, basic circuit principles are employed to easily create the analog model used in the emulated power system as opposed to phase shifting transformer modeling [6, 7] where augmented current injection techniques that lead to increased matrix sizes or their manipulation are used.

This paper is presented in the following manner. The next section gives an overview on the analog computation scheme followed by a section presenting the details of the phase-shifting transformer model. Simulation results in an emulation environment are then shown to verify the functionality of the transformer model.

II. DC Computational Methodology and Implementation

The complex current flowing in any branch or transmission line is a result of interaction between the bus or node voltages and transmission line impedance. Using admittance for simplicity and expressed in rectangular form, we have;

$$\begin{aligned} I &= Y \cdot V \\ &= \underbrace{\left(\underbrace{Y_{\mathrm{Re}}V_{\mathrm{Re}}}_{\text{Network I}} - \underbrace{Y_{\mathrm{Im}}V_{\mathrm{Im}}}_{\text{Network II}}\right)}_{I_{\mathrm{Re}}} + j\underbrace{\left(\underbrace{Y_{\mathrm{Im}}V_{\mathrm{Re}}}_{\text{Network III}} + \underbrace{Y_{\mathrm{Re}}V_{\mathrm{Im}}}_{\text{Network IV}}\right)}_{I_{\mathrm{Im}}} \end{aligned} \tag{1}$$

where subscripts Re and Im represent real and imaginary parts, respectively.

Four separate networks are required to emulate the complex current (1) flowing on any branch. Using the approach proposed by [2], four separate dc-resistive networks that represent the AC power system network can be used with the assumption that changes in angular frequency in the system are very small and considered as constant. Generators are represented as DC voltage sources, the transmission lines as resistive networks whose size is relative to line parameters and the loads sink current from the networks. The generators excite the networks with real and imaginary DC voltage components and the states (voltages and currents) of the resistive network provide the steady-state AC power flow solution.

Considering an arbitrary bus i with its complex voltage value, four networks with four buses are considered as equivalents: i^{I}, i^{II}, i^{III}, i^{IV}. For the described computational methodology to yield feasible and accurate values, the following conditions must be satisfied:

$$\left.\begin{aligned} V_{i^{I}} &= V_{i^{III}} = V_{i_{\mathrm{Re}}} \\ V_{i^{II}} &= V_{i^{IV}} = V_{i_{\mathrm{Im}}} \end{aligned}\right\} \tag{2}$$

III. Circuit Model of the Phase-Shifting Transformer Model

The circuit representation of an ideal transformer with turns ratio 1:t and a series impedance is shown in Figure 1[8].

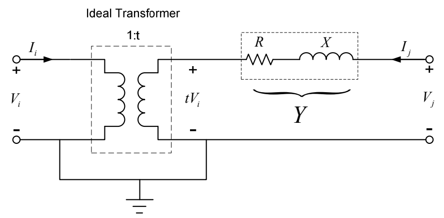

Figure 1. Circuit Representation of a Transformer

Developing equations for the voltages and currents in the circuit yields the following:

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} |t|^2 Y & -t^*Y \\ -tY & Y \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} \tag{3}$$

A pi equivalent circuit, shown in Figure 2, can be developed for a tap changing transformer from (3). If phase shifting is present the off diagonal entries of (3) become unequal and the existing circuit form will no longer be realizable.

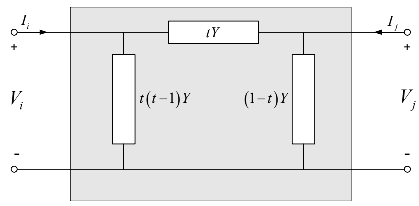

Figure 2. Equivalent circuit of a Tap Chaning Transformer

To account for phase shifting, where t is now defined as $t = |t|\angle\phi$, two dependent active sources have been placed in the transformer model and described by equations (4) - (6). These equations introduce the cross coupling dependency of the active sources in the model which describes the transformer's phase adjustment effect due to changes in the system.

$$\begin{aligned} V_{j'} &= V_j\angle\phi \\ &= \underbrace{\left(V_j^{\mathrm{Re}}\cos\phi - V_j^{\mathrm{Im}}\sin\phi\right)}_{V_{j'}^{\mathrm{Re}}} + j\underbrace{\left(V_j^{\mathrm{Im}}\cos\phi + V_j^{\mathrm{Re}}\sin\phi\right)}_{V_{j'}^{\mathrm{Im}}} \end{aligned} \tag{4}$$

$$\begin{aligned} V_{Th} &= \frac{I_j}{(1-t)Y} \\ &= \underbrace{\frac{1}{1-|t|}\left(I_j^{\mathrm{Re}}R - I_j^{\mathrm{Im}}X\right)}_{V_{Th}^{\mathrm{Re}}} + j\underbrace{\frac{1}{1-|t|}\left(I_j^{\mathrm{Re}}X + I_j^{\mathrm{Im}}R\right)}_{V_{Th}^{\mathrm{Im}}} \end{aligned} \tag{5}$$

$$\begin{aligned} I_j &= I_{j'}\angle-\phi \\ &= \underbrace{\left(I_{j'}^{\mathrm{Re}}\cos\phi + I_{j'}^{\mathrm{Im}}\sin\phi\right)}_{I_j^{\mathrm{Re}}} + j\underbrace{\left(I_{j'}^{\mathrm{Im}}\cos\phi - I_{j'}^{\mathrm{Re}}\sin\phi\right)}_{I_j^{\mathrm{Im}}} \end{aligned} \tag{6}$$

In order to be used in the DC emulation this equivalent circuit of the transformer was changed into rectangular coordinates similar to (1). The implemented circuit oriented model of the phase-shifting transformer used for emulation with four DC networks is shown in Figure 3. Sizing of the resistive elements in the model is done in same manner as in [4].

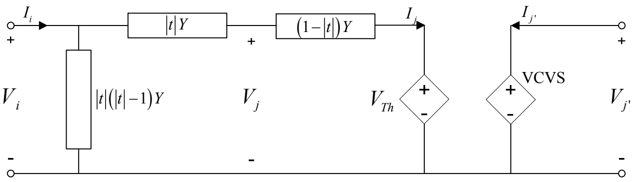

Figure 3. Circuit Model of a Phase-Shifting Transformer

IV. Model Testing and Results

The Analog Behavioral Models (ABM) and analog devices in Pspice serve as building blocks of electric circuit equivalent mathematical equations and scaled-model representation that describe the states and behavior of real power systems under various conditions. This approach is suitable for verifying the design and functionality of the computational scheme before designing an analog hardware. ABMs are used to develop and test the phase-shifting transformer model in a power system. To emulate the test system shown in Figure 4, four independent networks are used. Steady-state conditions of the system are studied under different transformer (tap and phase) settings and loading conditions.

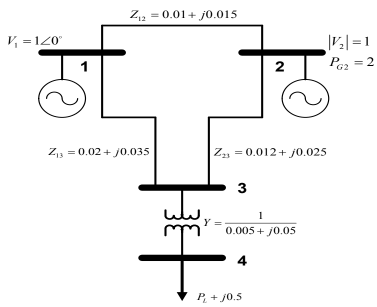

Figure 4. 4-Bus Test System

A. Component Modeling

Generators are represented as DC voltage sources at specified buses (bus 1 and 2). A generator circuit module is required for each and every generator in the system being emulated. The generator module [9] used in this paper contains circuitry for solving the swing equation and determines the complex voltage output for a generator. Its model is acceptable for transient stability analysis and steady-state representation.

A load model is a mathematical representation of the relationship between a bus voltage (magnitude and frequency) and the power (active and reactive) or current flowing into the load bus. In this paper an emulator load module that is based on a differential form of constant PQ-Load model [10] is used. The loads are represented by frequency and voltage dependent models, i.e.,

$$\left.\begin{aligned} P_L + K_{p\theta}\dot{\theta} &= P_e \\ Q_L + K_{qv}\dot{V} &= Q_e \end{aligned}\right\} \tag{7}$$

The first order integration of (7) yields

$$\left.\begin{aligned} \theta &= \frac{1}{K_{p\theta}} \int \left(-P_L + P_e\right) dt \\ |V| &= \frac{1}{K_{qv}} \int \left(-Q_L + Q_e\right) dt \end{aligned}\right\} \tag{8}$$

The load bus voltage magnitude is very sensitive to the difference in the reactive load and reactive electrical power as the load bus phase angle is very sensitive to the difference in the real load and real electrical power. The controllable exponential recovery rate of angle and voltage magnitude $K_{p\theta}$ and K_{qv} will dictate how rapidly the transient behavior described by (7) and (8) will settle to a steady-state value. For the analog computation scheme used here, the load module is very important. It attains feasible and measurable bus voltages at buses without generators or loads, i.e., bus 3 in the test system presented here. Floating nodes with specified values of $P_L = 0$ and $Q_L = 0$ should use the constant PQ load module to solve for their specific bus voltage in this computation scheme.

In the phase-shifting transformer model, active sources voltage values are obtained by a calculator whose inputs are measured signals in the system. The cross coupling dependency describe by (4) – (6) represent the transformer's phase adjustment due to changes in the system and allow for bidirectional flow of power. Figure 5 presents the configuration of ABM blocks in PSpice used to calculate real and imaginary voltages in (4).

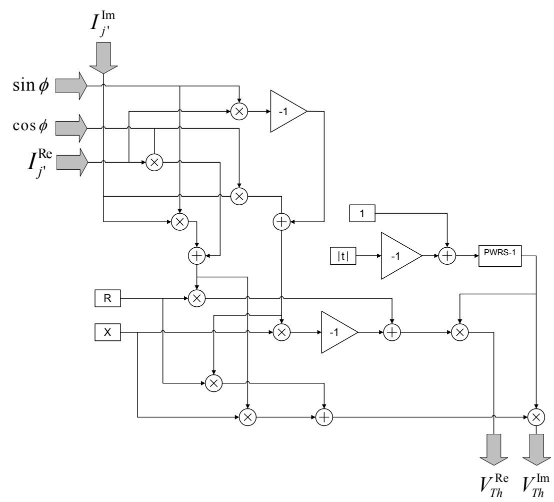

Figure 5. Configuration of Pspice ABM calculator for V_{Th}

In summary, all the components of the test power system are modeled using general purpose analog components. That is, the mathematical equations that describe the state of these components of real power system are represented using electrical circuit equivalents and ABMs. The network transmission lines [11] and transformer's resistive elements are represented as resistive networks whose size is relative to their parameters and the interconnection of these components will give a complete representation of the test power system.

B. Emulation and Simulation Comparison

To validate the model, different transformer's tap and phase settings in the test power system are studied under heavy active loading conditions and voltage magnitudes at the floating bus examined. The sample test system was constructed in PowerWorld [12] software and power flow results tabulated for benchmarking purposes. The same test system was represented in PSpice utilizing the DC emulation technique and compared favorably to the solutions obtained in PowerWorld simulations as shown in Table II.

TABLE I. VOLTAGES AT BUS 3

		V_3	
$\lvert t \rvert \angle \phi$	P_L	***PowerWorld***	***PSpice***
$0.9 \angle -30$	4.3	$0.8992 \angle -3.0130$	$0.8991 \angle -3.0129$
$1.05 \angle 30$	5.55	$0.8709 \angle -4.5433$	$0.8710 \angle -4.5433$

Automatic control was disabled in PowerWorld and for comparison the emulated system was used to obtain power flow solutions and validation of the accuracy of the model. This paper also addressed the limitations of [2]; the DC emulation technique could not be implemented to a full expanded (or unreduced) network where feasible and measurable voltage values at nodes without generator could not be obtained. Figure 6 shows the transient behavior of the real voltages in networks I and III and imaginary voltages in network II and IV at bus 3 for a specific loading condition and tap and phase settings on the transformer model. Note that conditions in (2) are satisfied.

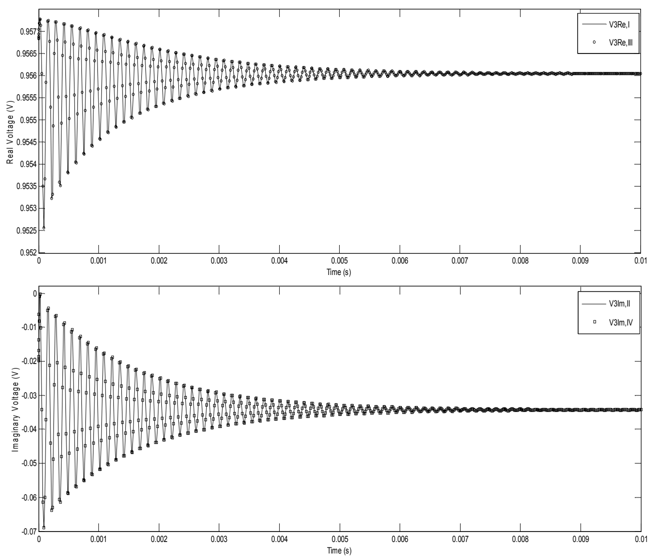

Figure 6. Transient Behavior of V_3^{Re} and V_3^{Im}

As alluded to earlier, the rates at which these voltages settle to a steady-state value depend on the recovery rate of angle and voltage magnitude in (7) and (8). These dynamic values enable load response studies in the analog computation scheme which only dynamic packages can provide, making the analog computation scheme a platform suitable for both dynamic and steady-state studies.

V. CONCLUSION

Theoretical development of a phase shifting transformer model has been presented and used for a previously proposed analog power flow technique. Results compare favorably to a commercially available power flow solver. Implementation of the model in analog hardware and development of a phase angle regulator that enables the use of the model for dynamic and steady-state studies of power systems in the analog emulation scheme is left for future work.

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