3.2.3. Propriedades da transformada $\mathcal{Z}$.

(Assume-se que as seq. têm amostras nulos para $k<0$) → (transformada unilateral)

A) Linearidade

Seja $F_1(z) = \mathcal{Z}\{f_1(k)\}$ e $F_2(z) = \mathcal{Z}\{f_2(z)\}$

e α e β reais

$$\mathcal{Z}\{\alpha f_1(k) + \beta f_2(k)\} = \alpha F_1(z) + \beta F_2(z)$$

- Demonstração:

$$\mathcal{Z}\{\alpha f_1(k) + \beta f_2(k)\} = \sum_{k=0}^{\infty} (\alpha f_1(k) + \beta f_2(k)) z^{-k} =$$

$$= \sum_{k=0}^{\infty} \alpha f_1(k) z^{-k} + \sum_{k=0}^{\infty} \beta f_2(k) z^{-k}$$

$$= \alpha \sum_{k=0}^{\infty} f_1(k) z^{-k} + \beta \sum_{k=0}^{\infty} f_2(k) z^{-k} = \alpha \mathcal{Z}\{f_1(k)\} + \beta \mathcal{Z}\{f_2(k)\}$$

$$= \alpha F_1(z) + \beta F_2(z)$$

B) DESLOCAMENTO

* AVANÇO: $\mathcal{Z}\{f(k+j)\} = z^j F(z) - \sum_{l=0}^{j-1} f(l) z^{j-l} \qquad (j>0)$

- DEMONSTRAÇÃO: $F(z) = \mathcal{Z}\{f(k)\}$

$$\mathcal{Z}\{f(k+j)\} = \sum_{k=0}^{\infty} f(k+j) z^{-k} \qquad (k+j=l)$$

$$= \sum_{l=j}^{\infty} f(l) z^{-l+j} = z^j \left[\sum_{l=j}^{\infty} f(l) z^{-l} \right]$$

$$= z^j \left[\sum_{l=0}^{\infty} f(l) z^{-l} - \sum_{l=0}^{j-1} f(l) z^{-l} \right]$$

$$\mathcal{Z}\{f(k+j)\} = z^{j}\sum_{\ell=0}^{\infty} f(\ell)\,z^{-\ell} - \sum_{\ell=0}^{j-1} f(\ell)\,z^{j-\ell}$$

$$\mathcal{Z}\{f(k+j)\} = z^{j}F(z) - \sum_{\ell=0}^{j-1} f(\ell)\,z^{j-\ell}$$

* ATRASO : $\mathcal{Z}\{f(k-j)\} = z^{-j}F(z)$ $\qquad (j>0)$

- DEMONSTRAÇÃO : $F(z) = \mathcal{Z}\{f(k)\}$

$$\mathcal{Z}\{f(k-j)\} = \sum_{k=0}^{\infty} f(k-j)\,z^{-k} \qquad (\ell = k-j)$$

$$= \sum_{\ell=-j}^{\infty} f(\ell)\,z^{-\ell-j} = z^{-j}\left[\sum_{\ell=-j}^{\infty} f(\ell)\,z^{-\ell}\right]$$

$$= z^{-j}\left[\underbrace{\sum_{\ell=-j}^{-1} f(\ell)\,z^{-\ell}}_{0} + \underbrace{\sum_{\ell=0}^{\infty} f(\ell)\,z^{-\ell}}_{F(z)}\right]$$

$$\mathcal{Z}\{f(k-j)\} = z^{-j}F(z)$$